```
# Industrial Organization II - Problem Set 1
# Demand Estimation (Logit)
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# Preliminaries -------------------------------------------------------------
rm(list = ls())
pacman :: p_load('data.table','dplyr','bbmle','AER','gmm','stargazer', 'optimx')
dir <- 'C:/Users/Gabriel Gonzalez Sut/Google Drive/PhD Sustainable - Columbia/'
course <- 'Sem 5 - Industrial Organization II/Problem Sets/'
ps <- 'PS1'
setwd(paste0(dir,course,ps))

# Import Data ---------------------------------------------------------------------------------------
auto <-  read.delim('ps1_blp_data_no_header.txt', header = FALSE)
colnames(auto) <- c('price','quantity','weight','hp','AC','firm')
auto$price <- as.numeric(auto$price)
auto$quantity <- as.numeric(auto$quantity)
auto$weight <- as.numeric(auto$weight)
auto$hp <- as.numeric(auto$hp)
auto$AC <- as.numeric(auto$AC)
auto$firm <- as.numeric(auto$firm)
M <- 100 * 10^6 # Market Size
lambda <- 4 * 10^(-6)

#  ---------------------------------------- Logit Model ------------------------------------------------------

logit <- as.data.table(auto)
logit <- logit[order(price),]
logit[, share := (quantity/M)] # Define share
logit[, delta := log(share) - log(1 - sum(logit$share))] # Define delta
variables <- c('weight','hp','AC')

# Functions
BLP.z <- function(data,covariates){
  n <- (length(covariates)*2)
  Z <- matrix(0,nrow(data),n)
  Z[,1:3] <- as.matrix(data[,covariates, with = FALSE])
  for (i in 1:nrow(data)){
    competitor <- as.matrix(data[firm != data$firm[i],covariates, with = FALSE])
    if (nrow(competitor) > 1) Z[i,4:6] <- colSums(competitor)
    if (nrow(competitor) == 1) Z[i,4:6] <- competitor
  }
  Z <- as.data.table(Z)
  colnames(Z) <- paste0('Z',1:n)
  return(Z)
}

Ownership <- function(data){
  H <- matrix(0,nrow(data),nrow(data))
  for (i in 1:nrow(data)){
    for (j in 1:nrow(data)){
      if(data$firm[i] == data$firm[j]) H[i,j] <- 1
    }
  }
  return(H)
}

derivatives <- function(data){
  gamma <- matrix(0,nrow(data),nrow(data))
  for (i in 1:nrow(data)){
    for (j in 1:nrow(data)){
      if(i == j){
        gamma[i,j] <- -(data$share[i] -data$share[i]^2)
      }else{
        gamma[i,j] <- data$share[i] * data$share[j]
      }
    }
  }
  return(gamma)
}

################ Demand Only (2SLS/Effiecient GMM) ##################

BLP.instruments <- BLP.z(logit,variables)
logit$n.price <- -logit$price
logit <- cbind(logit, BLP.instruments)
attach(logit)

# 2SLS
mod6 <- ivreg(delta ~ -1 + n.price + weight + hp + AC | -1 + Z1 + Z2 + Z3 + Z4 + Z5 + Z6, data = logit)
mod6b <- ivreg(delta ~ n.price + weight + hp + AC | Z1 + Z2 + Z3 + Z4 + Z5 + Z6, data = logit)

summary(mod6b)

# GMM
Z <- BLP.instruments
X <- cbind(n.price,logit[,variables, with = FALSE])

moment <- function(theta){
  xi <- delta - as.matrix(X[,1:4]) %*% as.matrix(theta[1:4])
  g <- (t(Z) %*% xi)*(1/nrow(X))
  W <- as.matrix(t(Z)) %*% as.matrix(Z)
  obj <- t(g) %*% solve(W) %*% g
}
gmm.mod6 <- opm(par = c(0,0,0,0), fn = moment, method=c('BFGS'))

Z <- cbind(rep(1,131),BLP.instruments)
X <- cbind(rep(1,131),n.price,logit[,variables, with = FALSE])

moment <- function(theta){
  xi <- delta - as.matrix(X[,1:5]) %*% as.matrix(theta[1:5])
  g <- (t(Z) %*% xi)*(1/nrow(X))
  W <- as.matrix(t(Z)) %*% as.matrix(Z)
  obj <- t(g) %*% solve(W) %*% g
}
gmm.mod6b <- opm(par = c(0,0,0,0,0), fn = moment, method=c('BFGS'))
```

```
# Results
mod6$coefficients
cbind(gmm.mod6$p1,gmm.mod6$p2,gmm.mod6$p3,gmm.mod6$p4)
mod6b$coefficients
cbind(gmm.mod6b$p1,gmm.mod6b$p2,gmm.mod6b$p3,gmm.mod6b$p4,gmm.mod6b$p5)
stargazer(mod6,mod6b, title="Just Demand")

################ Demand + Supply (GMM) #################

# Constant
H <- Ownership(logit)
gamma <- derivatives(logit)
Z <- cbind(rep(1,131),BLP.instruments)
Z1 <- rbind(as.matrix(Z),matrix(0,nrow(Z),ncol(Z)))
Z2 <- rbind(matrix(0,nrow(Z),ncol(Z)),as.matrix(Z))
inst <- rbind(t(Z1),t(Z2))
Xd <- cbind(rep(1,131), logit[,variables, with = FALSE])
X <- cbind(rep(1,131), logit[,variables, with = FALSE])

moment <- function(theta){
  alpha <- theta[1]
  beta <- theta[2:5]
  eta <- theta[6]
  gamma_s <- theta[7:10]
  omega <-  as.matrix(price) + solve(alpha * H * gamma) %*% as.matrix(share) -
    as.matrix(X) %*% as.matrix(gamma_s) - eta * as.matrix(quantity)
  xi <- as.matrix(delta) - as.matrix(Xd) %*% as.matrix(beta) + alpha * as.matrix(price)
  W <-  as.matrix(inst) %*% as.matrix(t(inst))
  error <- rbind(xi,omega)
  g <- (inst %*% error)*(1/nrow(X))
  obj <- t(g) %*% solve(W) %*% g
}
gmm.mod7<- opm(par = c(0.1,0,0,0,0,0,0,0,0,0), fn = moment, method=c('BFGS'))

# Results
results <- data.table(parameter = c('alpha', 'constant', 'beta1', 'beta2', 'beta3', 'eta','constant', 'gamma1',
                                    'gamma2', 'gamma3'),
                      value = c(gmm.mod7$p1,gmm.mod7$p2,gmm.mod7$p3,gmm.mod7$p4,gmm.mod7$p5,
                                gmm.mod7$p6,gmm.mod7$p7,gmm.mod7$p8,gmm.mod7$p9,gmm.mod7$p10))
results

# No Constant
H <- Ownership(logit)
gamma <- derivatives(logit)
Z <- BLP.instruments
Z1 <- rbind(as.matrix(Z),matrix(0,nrow(Z),ncol(Z)))
Z2 <- rbind(matrix(0,nrow(Z),ncol(Z)),as.matrix(Z))
inst <- rbind(t(Z1),t(Z2))
Xd <- cbind(logit[,variables, with = FALSE])
X <- cbind(logit[,variables, with = FALSE])

moment <- function(theta){
  alpha <- theta[1]
  beta <- theta[2:4]
  eta <- theta[5]
  gamma_s <- theta[6:8]
  omega <-  as.matrix(price) + solve(alpha * H * gamma) %*% as.matrix(share) -
    as.matrix(X) %*% as.matrix(gamma_s) - eta * as.matrix(quantity)
  xi <- as.matrix(delta) - as.matrix(Xd) %*% as.matrix(beta) + alpha * as.matrix(price)
  W <-  as.matrix(inst) %*% as.matrix(t(inst))
  error <- rbind(xi,omega)
  g <- (inst %*% error)*(1/nrow(X))
  obj <- t(g) %*% solve(W) %*% g
}
gmm.mod7b<- opm(par = c(0.1,0,0,0,0,0,0,0), fn = moment, method=c('BFGS'))

# Results
resultsb <- data.table(parameter = c('alpha', 'beta1', 'beta2', 'beta3', 'eta', 'gamma1',
                                     'gamma2', 'gamma3'),
                       value = c(gmm.mod7b$p1,gmm.mod7b$p2,gmm.mod7b$p3,gmm.mod7b$p4,gmm.mod7b$p5,
                                 gmm.mod7b$p6,gmm.mod7b$p7,gmm.mod7b$p8))
resultsb

################ Elasticities #################

elast.demand <- matrix(0,nrow(logit),nrow(logit))
elast.demand <- as.data.table(elast.demand)
for (i in 1:nrow(logit)){
  for (j in 1:nrow(logit)){
    if(i == j){
      elast.demand[i,j] <- -gmm.mod6b$p2 * logit$price[i] * (1 -logit$share[i])
    }else{
      elast.demand[i,j] <- gmm.mod6b$p2 * logit$price[j] * logit$share[j]
    }
  }
}

elast.supply <- matrix(0,nrow(logit),nrow(logit))
elast.supply <- as.data.table(elast.supply)
for (i in 1:nrow(logit)){
  for (j in 1:nrow(logit)){
    if(i == j){
      elast.supply[i,j] <- -gmm.mod7$p1 * logit$price[i] * (1 -logit$share[i])
    }else{
      elast.supply[i,j] <- gmm.mod7$p1 * logit$price[j] * logit$share[j]
    }
  }
}

logit$elasticity <- diag(as.matrix(elast.demand))

# Plots
png('logit_price1.png')
plot(price, diag(as.matrix(elast.demand)), xlab = "Car Price", ylab = "Price Elasticity")
dev.off()
```

```
png('logit_price2.png')
plot(price, diag(as.matrix(elast.supply)), xlab = "Car Price", ylab = "Price Elasticity")
dev.off()

elast.demand$price <- price
elast.supply$price <- price

png('logit_cross1.png')
par(mfrow=c(3,2))
plot(elast.demand$price[-c(1)], as.matrix(elast.demand[-c(1),1]),
     xlab = "Others Car's Price", ylab = "Cross-Price Elasticity", main = 'Car Priced $6488');
plot(elast.demand$price[-c(2)], as.matrix(elast.demand[-c(2),2]),
     xlab = "Others Car's Price", ylab = "Cross-Price Elasticity", main = 'Car Priced $8748');
plot(elast.demand$price[-c(3)], as.matrix(elast.demand[-c(3),3]),
     xlab = "Others Car's Price", ylab = "Cross-Price Elasticity", main = 'Car Priced $11588');
plot(elast.demand$price[-c(4)], as.matrix(elast.demand[-c(4),4]),
     xlab = "Others Car's Price", ylab = "Cross-Price Elasticity", main = 'Car Priced $12268');
plot(elast.demand$price[-c(5)], as.matrix(elast.demand[-c(5),5]),
     xlab = "Others Car's Price", ylab = "Cross-Price Elasticity", main = 'Car Priced $14898');
plot(elast.demand$price[-c(6)], as.matrix(elast.demand[-c(6),6]),
     xlab = "Others Car's Price", ylab = "Cross-Price Elasticity", main = 'Car Priced $21050');
dev.off()

png('logit_cross2.png')
par(mfrow=c(3,2))
plot(elast.supply$price[-c(1)], as.matrix(elast.supply[-c(1),1]),
     xlab = "Others Car's Price", ylab = "Cross-Price Elasticity", main = 'Car Priced $6488');
plot(elast.supply$price[-c(2)], as.matrix(elast.supply[-c(2),2]),
     xlab = "Others Car's Price", ylab = "Cross-Price Elasticity", main = 'Car Priced $8748');
plot(elast.supply$price[-c(3)], as.matrix(elast.supply[-c(3),3]),
     xlab = "Others Car's Price", ylab = "Cross-Price Elasticity", main = 'Car Priced $11588');
plot(elast.supply$price[-c(4)], as.matrix(elast.supply[-c(4),4]),
     xlab = "Others Car's Price", ylab = "Cross-Price Elasticity", main = 'Car Priced $12268');
plot(elast.supply$price[-c(5)], as.matrix(elast.supply[-c(5),5]),
     xlab = "Others Car's Price", ylab = "Cross-Price Elasticity", main = 'Car Priced $14898');
plot(elast.supply$price[-c(6)], as.matrix(elast.supply[-c(6),6]),
     xlab = "Others Car's Price", ylab = "Cross-Price Elasticity", main = 'Car Priced $21050')
dev.off()
par(mfrow=c(1,1))
```